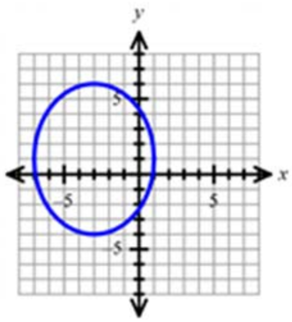
Math 127 **Ch 9 Review Worksheet** **Name**____________________________

Matching: For #1 – 9, match each equation to its graph. No item will be used more than once.

Equations:

1) $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{25} = 1$

G)



This is an **ellipse** because of the + sign between terms and the different denominators in the two terms.

The center is $(-3, 1)$ based on the numerators of the terms (change the signs of the numbers in parentheses).

The spread of the ellipse is based on the square roots of the denominators of the terms.

x-values spread out by $\sqrt{16} = 4$ left and right from the center.

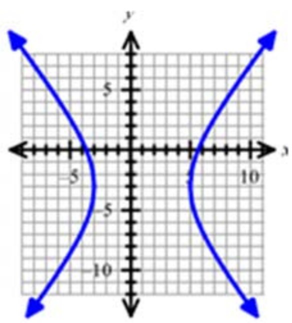
y-values spread out by $\sqrt{25} = 5$ up and down from the center.

Answers D and G are ellipses.

Answer G has a center at $(-3, 1)$ and the proper spread.

2) $\frac{(x-1)^2}{16} - \frac{(y+3)^2}{25} = 1$

F)



This is a **hyperbola** because of the − sign between terms.

It opens left and right because the x-term is positive.

The center is $(1, -3)$ based on the numerators of the terms (change the signs of the numbers in parentheses).

The spread of the hyperbola is based on the square roots of the denominators of the terms.

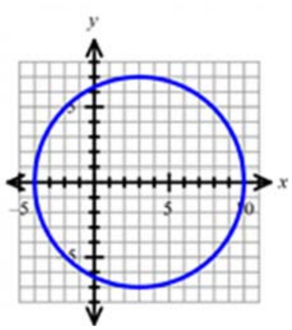
x-values spread out by $\sqrt{16} = 4$ left and right from the center.

Answers E and F are hyperbolas.

Answer F has a center at $(1, -3)$ and opens left and right.

3) $(x-3)^2 + y^2 = 49$

I)



This is a **circle** because of the + sign between terms, and no (or the same) denominators.

The center is $(3, 0)$ based on the x- and y- terms (change the signs of the numbers in parentheses).

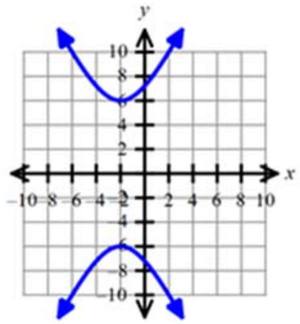
The radius of the circle is the square root of the right side of the equation. $r = \sqrt{49} = 7$.

Answers B and I are circles.

Answer I has a center at $(3, 0)$ and a radius of 7.

4) $\frac{y^2}{36} - \frac{(x+2)^2}{9} = 1$

E)



This is a **hyperbola** because of the $-$ sign between terms.

It opens up and down because the y-term is positive.

The center is $(-2, 0)$ based on the numerators of the terms (change the signs of the numbers in parentheses).

The spread of the hyperbola is based on the square roots of the denominators of the terms.

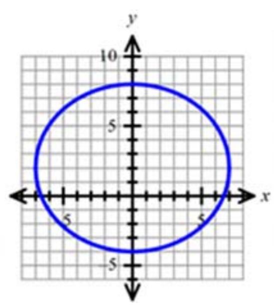
y-values spread out by $\sqrt{36} = 6$ up and down from the center.

Answers E and F are hyperbolas.

Answer E has a center at $(-2, 0)$ and opens up and down.

5) $\frac{x^2}{49} + \frac{(y-2)^2}{36} = 1$

D)



This is an **ellipse** because of the $+$ sign between terms and the different denominators in the two terms.

The center is $(0, 2)$ based on the numerators of the terms (change the signs of the numbers in parentheses).

The spread of the ellipse is based on the square roots of the denominators of the terms.

x-values spread out by $\sqrt{49} = 7$ left and right from the center.

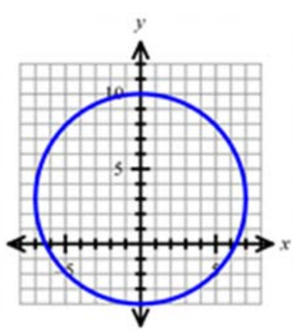
y-values spread out by $\sqrt{36} = 6$ up and down from the center.

Answers D and G are ellipses.

Answer D has a center at $(0, 2)$ and the proper spread.

6) $x^2 + (y-3)^2 = 49$

B)



This is a **circle** because of the $+$ sign between terms, and no (or the same) denominators.

The center is $(0, 3)$ based on the x- and y- terms (change the signs of the numbers in parentheses).

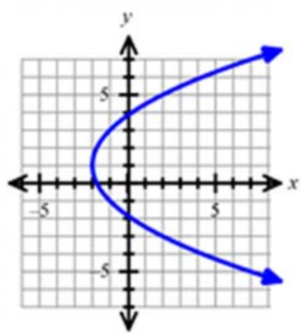
The radius of the circle is the square root of the right side of the equation. $r = \sqrt{49} = 7$.

Answers B and I are circles.

Answer B has a center at $(0, 3)$ and a radius of 7.

7) $(y-1)^2 = 4(x+2)$

A)



This is a **parabola** because only one of the terms is squared.

The x- term has an exponent of 1 so the parabola opens in the x-direction (to the left or right).

The x- term is positive so the parabola opens in the positive direction (to the right).

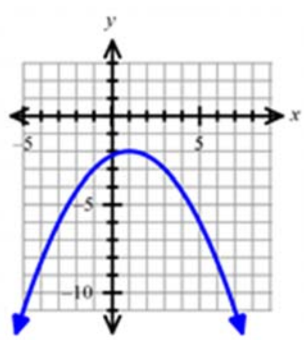
The vertex is $(-2, 1)$ based on the x- and y- terms (change the signs of the numbers in parentheses).

Answers A, C and H are parabolas.

Answer A has a vertex at $(-2, 1)$ and opens to the right.

8) $(x-1)^2 = -4(y+2)$

H)



This is a **parabola** because only one of the terms is squared.

The y- term has an exponent of 1 so the parabola opens in the y-direction (up or down).

The y- term is negative so the parabola opens in the negative direction (down).

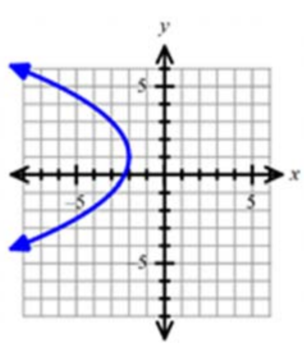
The vertex is $(1, -2)$ based on the x- and y- terms (change the signs of the numbers in parentheses).

Answers A, C and H are parabolas.

Answer H has a vertex at $(1, -2)$ and opens down.

9) $(y-1)^2 = -4(x+2)$

C)



This is a **parabola** because only one of the terms is squared.

The x- term has an exponent of 1 so the parabola opens in the x-direction (to the left or right).

The x- term is negative so the parabola opens in the negative direction (to the left).

The vertex is $(-2, 1)$ based on the x- and y- terms (change the signs of the numbers in parentheses).

Answers A, C and H are parabolas.

Answer C has a vertex at $(-2, 1)$ and opens to the left.

For #10 – 13, identify each conic as a circle, ellipse, hyperbola, or parabola.

10) $5x^2 - 8x + 10y^2 + 30y = 15$

Both x- and y-terms are squared and positive, with different coefficients, so this is an **ellipse**.

11) $x^2 + 3x = y^2 - 2y + 18$

If we put all of the variables on one side of the equal sign, the x- and y-terms are both squared and will have opposite signs, so this is a **hyperbola**.

12) $18y^2 + 5x = -18x^2 + 36$

If we put all of the variables on one side of the equal sign, the x- and y-terms are both squared and positive, with the same coefficient, so this is a **circle.**

13) $3x^2 - 2y = 9x + 11$

One of the variable terms is not squared, so this is a **parabola**.

For #14 – 16, Identify the type of conic; write in standard form; find the coordinates of the center and foci (if applicable), find the radius (if applicable), and graph on the provided coordinate system. **If needed, round to 3 decimal places.**

14) $4x^2 + 9y^2 + 16x = 20$

Both x- and y-terms are squared and positive, with different coefficients, so this is an **ellipse**.

We need to complete the squares for both the x- and y-terms.

$4x^2 + 9y^2 + 16x = 20$

$(4x^2 + 16x + \_\_\_\_) + 9y^2 = 20$

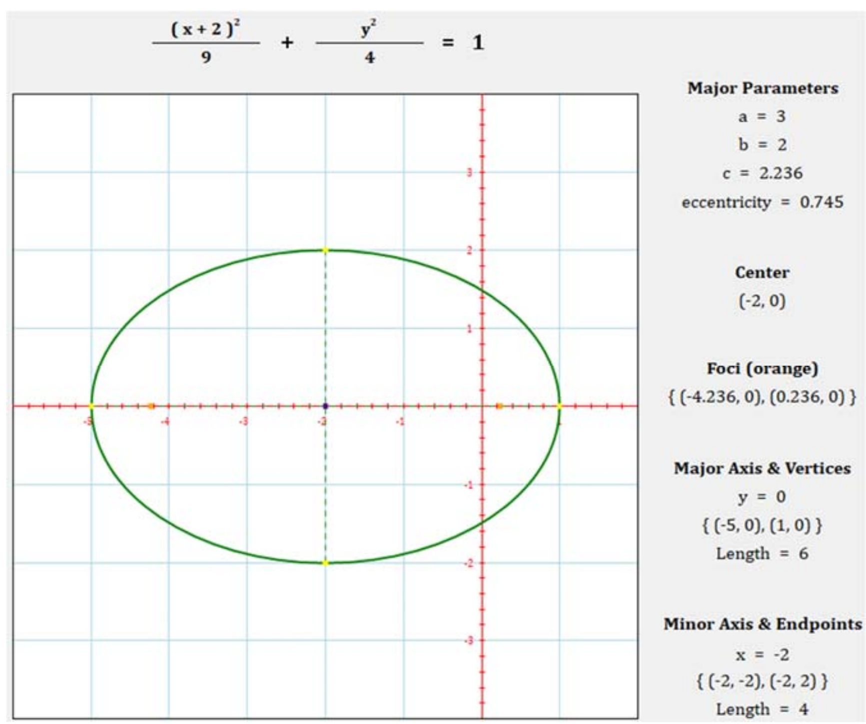
$4(x^2 + 4x + \_\_\_\_) + 9y^2 = 20$

$4(x^2 + 4x + 4) + 9y^2 = 20 + (4 \cdot 4)$

$4(x+2)^2 + 9y^2 = 36$

$\frac{4(x+2)^2}{36} + \frac{9y^2}{36} = 1$

$\mathbf{\frac{(x+2)^2}{9} + \frac{y^2}{4} = 1}$



This ellipse has **center** $(\mathbf{-2, 0})$ based on the numerators of the terms (change the signs of the numbers in parentheses).

The spread is $\sqrt{9} = 3$ in the x-direction.

The spread is $\sqrt{4} = 2$ in the y-direction.

The major axis is $y = 0$ (wide x) because the x-denominator is larger.

The foci are $\sqrt{9-4} = \sqrt{5} \sim 2.236$ away from the center along the major axis, i.e., in both x-directions (left and right).

So, the foci are: $(-2 - 2.236, 0) = (\mathbf{-4.236.0})$ and $(-2 + 2.236, 0) = (\mathbf{0.236.0})$.

15) $7x^2 - 14x + 7y^2 - 28y = 21$

Both x- and y-terms are squared and positive, with the same coefficients, so this is a **circle**.

We need to complete the squares for both the x- and y-terms.

$7x^2 - 14x + 7y^2 - 28y = 21$

$x^2 - 2x + y^2 - 4y = 3$

$(x^2 - 2x + \_\_\_\_) + (y^2 - 4y + \_\_\_\_) = 3$

$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 3 + 1 + 4$

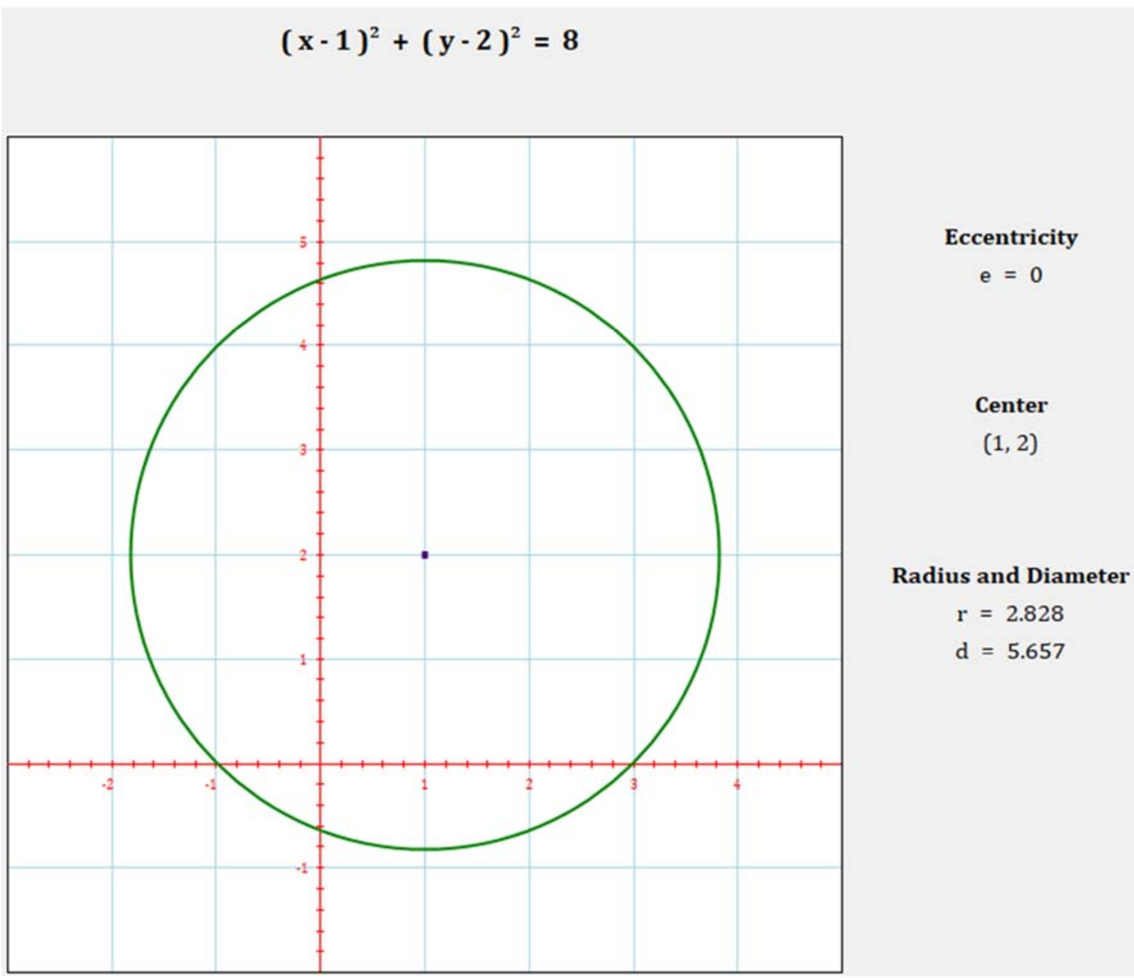
$(x-1)^2 + (y-2)^2 = 8$

$\mathbf{(x-1)^2 + (y-2)^2 = 8}$

This ellipse has **center** $(\mathbf{1}, \mathbf{2})$ based on the x- and y-terms (change the signs of the numbers in parentheses).

The radius is $r = \sqrt{8} \sim 2.828$.

There are no foci for a circle.



16) $121x^2 + 25y^2 - 150y - 2800 = 0$

Both x- and y-terms are squared and positive, with different coefficients, so this is an **ellipse**.

> Note: since the examples on the sample test are ellipses and circles, expect hyperbolas and parabolas on the real test. Be prepared!

We need to complete the squares for both the x- and y-terms.

$121x^2 + 25y^2 - 150y - 2800 = 0$

$121x^2 + (25y^2 - 150y + \_\_\_\_) = 2800$

$121x^2 + 25(y^2 - 6y + \_\_\_\_) = 2800$

$121x^2 + 25(y^2 - 6y + 9) = 2800 + (25 \cdot 9)$

$121x^2 + 25(y - 3)^2 = 3025$

$\frac{121x^2}{3025} + \frac{25(y-3)^2}{3025} = 1$

$\mathbf{\frac{x^2}{25} + \frac{(y-3)^2}{121} = 1}$

This ellipse has **center** $(\mathbf{0}, \mathbf{3})$ based on the numerators of the terms (change the signs of the numbers in parentheses).

The spread is $\sqrt{25} = 5$ in the x-direction.

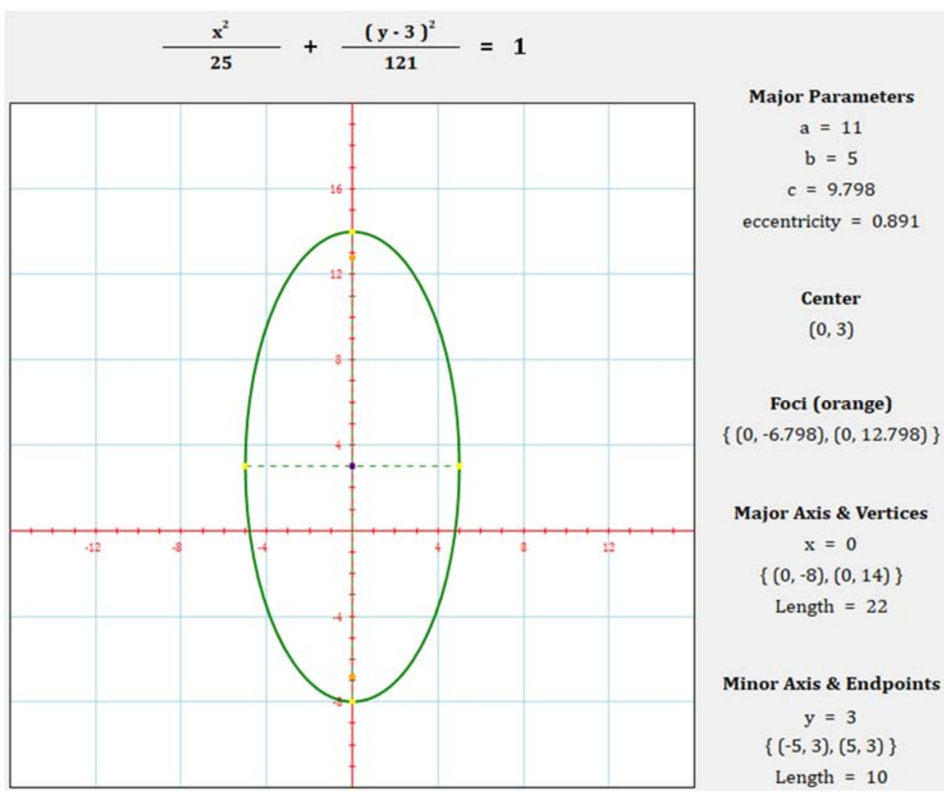
The spread is $\sqrt{121} = 11$ in the y-direction.

The major axis is on the line $x = 0$ (long in the y direction) because the y-denominator is larger.

The foci are $\sqrt{121 - 25} = \sqrt{96}$ ~ 9.798 away from the center along the major axis, i.e., in both y-directions (up and down).

So, the **foci** are:

$(0, 3 - 9.798) = (\mathbf{0}, \mathbf{-6.798})$,
$(0, 3 + 9.798) = (\mathbf{0}, \mathbf{12.798})$.



For #17 – 18: For each parabola given, write in standard form, find the coordinates of the vertex and focus, write the equation of the directrix, find the length of the latus rectum, and graph on the provided coordinate system. As needed, round to 3 decimal places.

17) $y^2 - 12y - 12x = 0$

Graphing parabolas is relatively easy compared to graphing ellipses or hyperbolas. The key to graphing a parabola is to identify its vertex and orientation (which way it opens). Consider the form of the above equation:

$$(y-k)^2 = 4p(x-h)$$

$$y^2 - 12y - 12x = 0$$

$$y^2 - 12y + 36 = 12x + 36$$

$$(y-6)^2 = 12(x+3)$$

From this equation, we can determine the following:

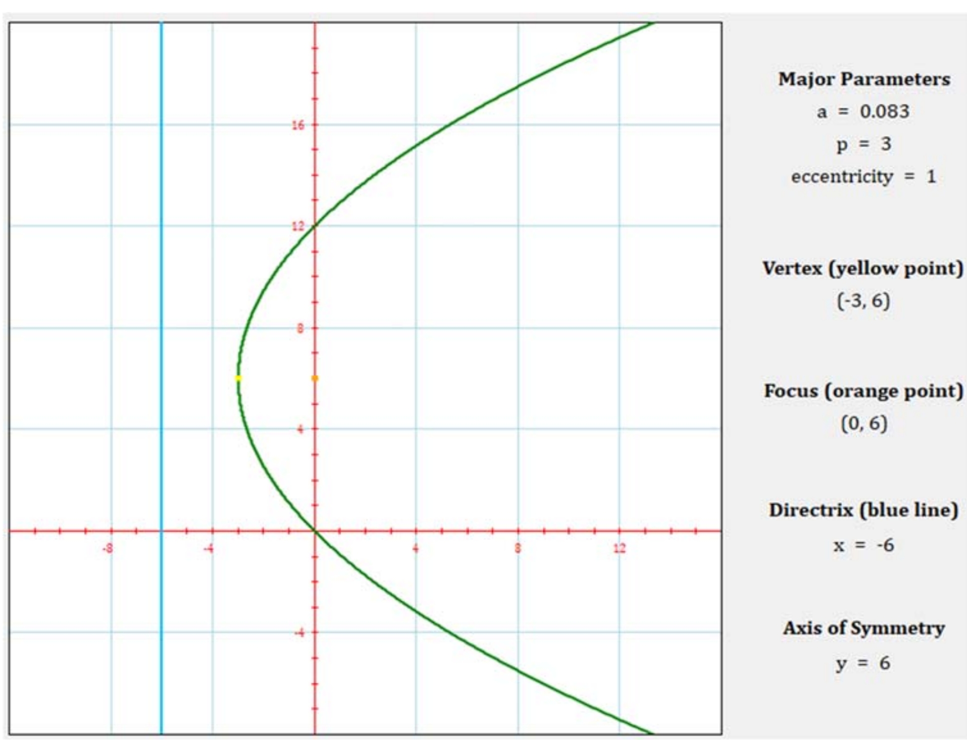
- The **vertex** of the parabola is $(h,k) = (-3, 6)$.
- Since the y-term is squared, the parabola has a vertical Directrix (i.e., it opens left or right).
- The **length of the latus rectum** is $|4p| = 12$. $p = 3$ is positive, so the parabola opens to the right.
- Since the y-term is squared, the **focus** is located $p = 3$ units to the right of the vertex, $(-3, 6)$: $(0, 6)$.
- The **equation of the directrix** is: $x = h - p \quad \Rightarrow \quad x = -3 - 3 \quad \Rightarrow \quad x = -6$

Let's find a couple of points to help us draw our graph of the parabola. Rewrite the equation in a simpler form to find x, given y.

$$x = \frac{1}{12}(y-6)^2 - 3$$

We already have a point – the vertex, at $(-3, 6)$. Let's find a couple more:

- Let $y = 0$. Then $x = \frac{1}{12}(0-6)^2 - 3 = 0$. This gives us the point $(0, 0)$.
- Let $y = 12$. Then $x = \frac{1}{12}(12-6)^2 - 3 = 0$. This gives us the point $(0, 12)$.



18) $3x^2 + 24x + 15y + 3 = 0$

The key to graphing a parabola is to identify its vertex and orientation (which way it opens). Consider the form of the above equation:

$$(x - h)^2 = 4p(y - k)$$

$$3x^2 + 24x + 15y + 3 = 0$$

$$x^2 + 8x + 5y + 1 = 0$$

$$(x^2 + 8x + \_\_\_\_) = -5y - 1$$

$$(x^2 + 8x + 16) = -5y - 1 + 16$$

$$\boldsymbol{(x + 4)^2 = -5(y - 3)}$$

From this equation, we can determine the following:

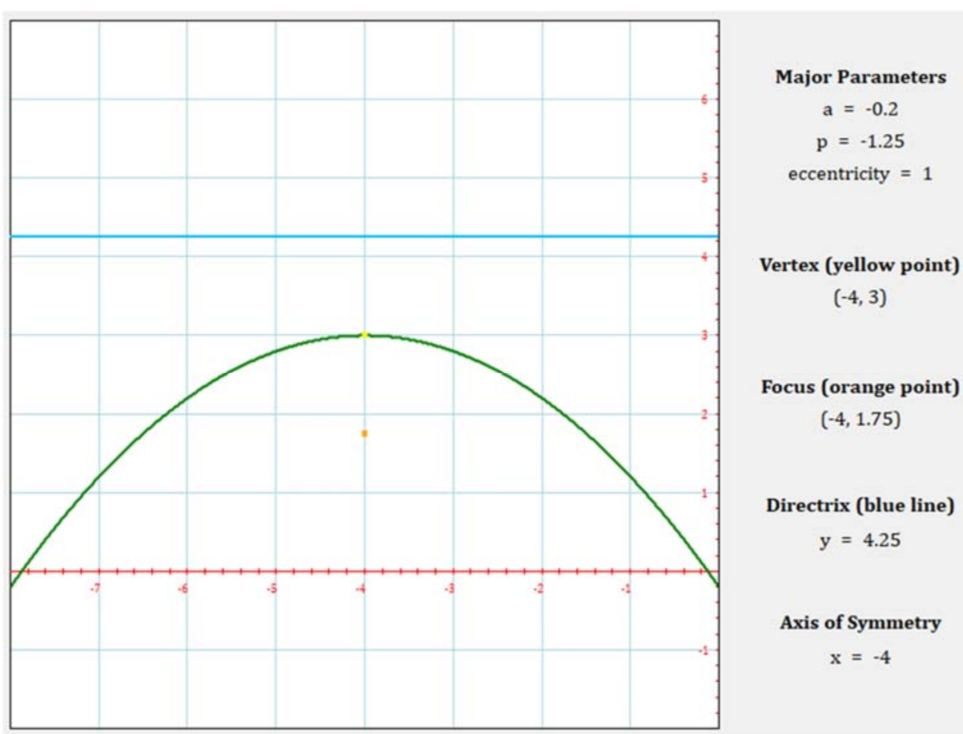
- The **vertex** of the parabola is $(h, k) = (\mathbf{-4, 3})$.
- Since the x-term is squared, the parabola has a horizontal Directrix (i.e., it opens up or down).
- The **length of the latus rectum** is $|4p| = \mathbf{5}$. $p = -5/4$ is negative, so the parabola opens down.
- Since the x-term is squared, the **focus** is located $5/4$ units to the below the vertex, $(-4, 3)$: $(\mathbf{-4, 1.75})$.
- The **equation of the directrix** is: $y = k - p \quad \Rightarrow \quad y = 3 - \left(-\frac{5}{4}\right) \quad \Rightarrow \quad \mathbf{y = 4.25}$

Let's find a couple of points to help us draw our graph of the parabola. Rewrite the equation in a simpler form to find y, given x.

$$y = -\frac{1}{5}(x + 4)^2 + 3$$

We already have a point – the vertex, at $(-4, 3)$. Let's find a couple more:

- Let $x = 0$. Then $y = -\frac{1}{5}(x + 4)^2 + 3 = -\frac{1}{5}$. This gives us the point $\left(0, -\frac{1}{5}\right)$.
- Let $x = -8$. Then $y = -\frac{1}{5}(x + 4)^2 + 3 = -\frac{1}{5}$. This gives us the point $\left(-8, -\frac{1}{5}\right)$.



19) Consider the hyperbola given. Write the equation in standard form, find the coordinates of the center and foci, write the equations of the asymptotes in (h, k) form, and graph on the provided coordinate system. $36(y-3)^2 - 9(x+2)^2 = 324$. If needed, round to 3 decimal places.

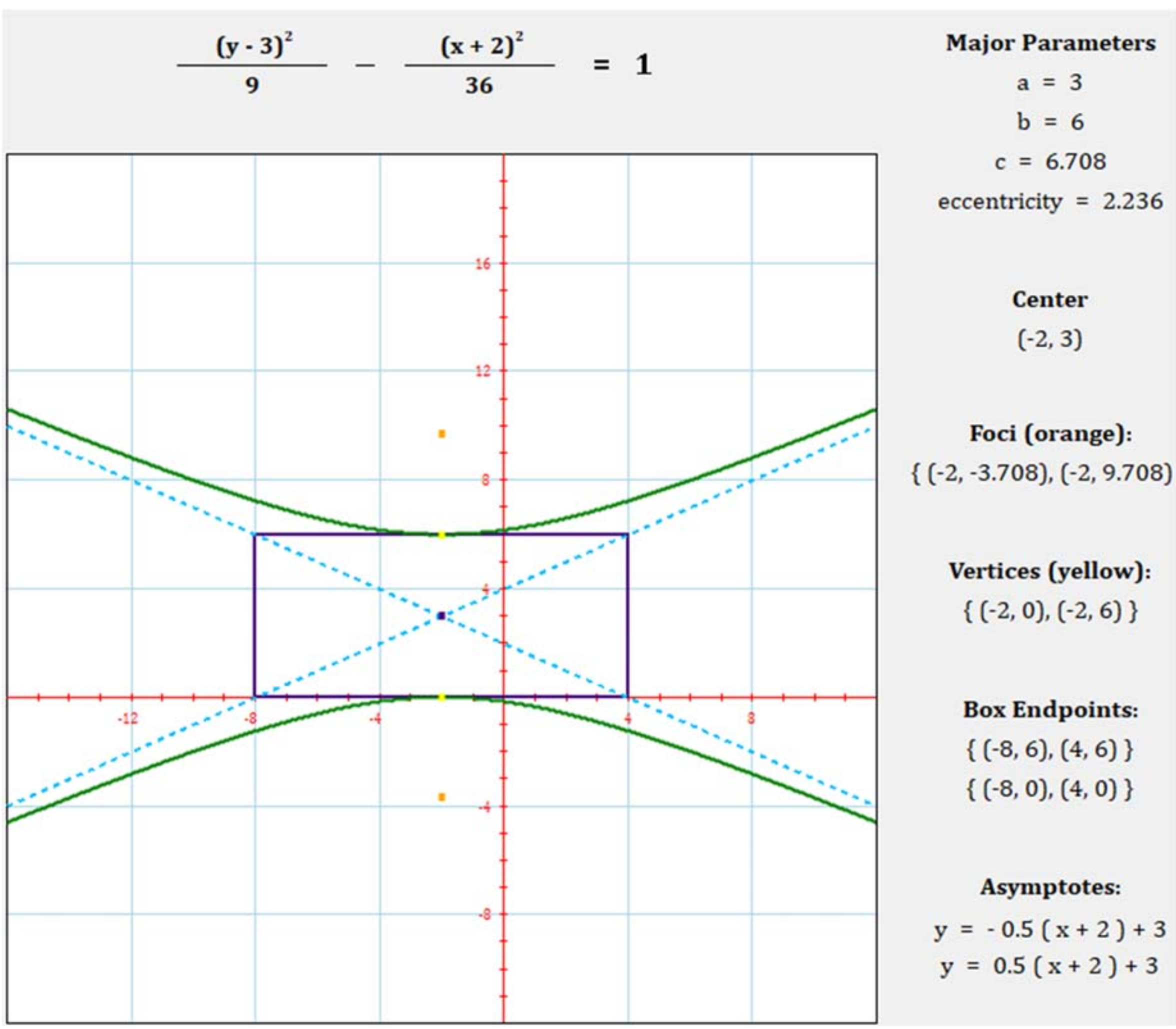
Divide by 324 to get this equation in standard form:

$\frac{(y-3)^2}{9} - \frac{(x+2)^2}{36} = 1$ Standard form is: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Since the y-term is positive, we know that the hyperbola has a vertical transverse axis.

- $h = -2, \ k = 3$
- $a^2 = 9$, so $a = 3$, $b^2 = 36$, so $b = 6$, $c^2 = a^2 + b^2 = 9 + 36 = 45$, so $c = \sqrt{45} \sim 6.708$
- **The center is** (h, k): $(-2, 3)$
- The center, vertices, and foci lie on the vertical transverse axis (VTA). On a VTA:
  The vertices are a units up and down from the center: $(-2, 3 \pm 3) \Rightarrow \{(-2, 0), (-2, 6)\}$
  The foci are c units up and down from the center: $\left(-2, 3 \pm \sqrt{45}\right) \Rightarrow \{(-2, -3.708), (-2, 9.708)\}$
- The asymptotes for a hyperbola with a vertical transverse axis are $y = \pm\frac{a}{b}(x-h) + k$. So:
  The asymptotes are: $y = \pm\frac{a}{b}(x-h) + k \Rightarrow y = \pm\frac{1}{2}(x+2) + 3$

To graph the hyperbola, first graph the asymptotes and the vertices, and then sketch in the rest.



20) Consider the hyperbola given. Write the equation in standard form, find the coordinates of the center and foci, write the equations of the asymptotes in (h, k) form, and graph on the provided coordinate system. $4(x-1)^2 - 25y^2 = 100$. If needed, round to 3 decimal places.

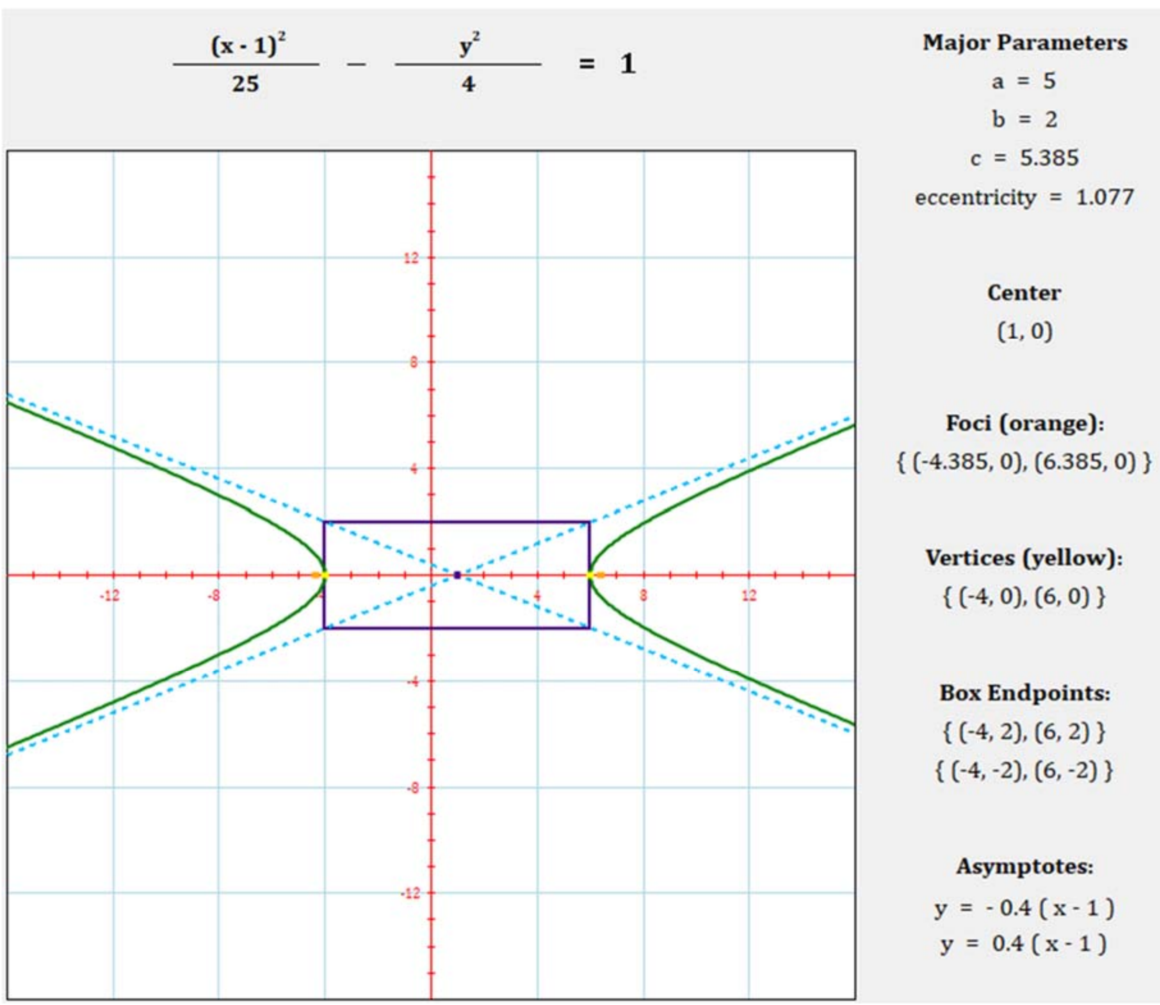
Divide by 100 to get this equation in standard form:

$\frac{(x-1)^2}{25} - \frac{y^2}{4} = 1$ Standard form is: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Since the x-term is positive, we know that the hyperbola has a horizontal transverse axis.

- $h = 1, \ k = 0$
- $a^2 = 25$, so $a = 5$, $b^2 = 4$, so $b = 2$, $c^2 = a^2 + b^2 = 25 + 4 = 29$, so $c = \sqrt{29} \sim 5.385$
- **The center is** (h, k): $(\mathbf{1}, \mathbf{0})$
- The center, vertices, and foci lie on the horizontal transverse axis (HTA). On an HTA:
  - The vertices are a units left and right from the center: $(1 \pm 5, 0) \Rightarrow \{(-4, 0), (6, 0)\}$
  - **The foci are** c units left and right from the center: $(1 \pm \sqrt{29}, 0) \Rightarrow \{(\mathbf{-4.385}, \mathbf{0}), (\mathbf{6.385}, \mathbf{0})\}$
- The asymptotes for a hyperbola with a horizontal transverse axis are $y = \pm\frac{b}{a}(x-h) + k$. So:
  - **The asymptotes are:** $y = \pm\frac{b}{a}(x-h) + k \Rightarrow y = \pm\frac{2}{5}(x-1)$

To graph the hyperbola, first graph the asymptotes and the vertices, and then sketch in the rest.



For #21 – 24, write the equation in standard form that meets the given requirements.

21) Ellipse; foci: $(5,0),(-5,0)$; y –intercepts: $(0,3),(0,-3)$

This ellipse has foci $(\pm 5,0)$, and therefore has a horizontal major axis.

The standard form for an ellipse with a horizontal major axis is:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

The center of the ellipse is at the midpoint of the foci and the y-intercepts.

- $h=\frac{5+(-5)}{2}=0,\ k=\frac{3+(-3)}{2}=0$

The foci are located c units left and right from the center, so $c=5-0=5.$

The values of a and b can be determined from the foci and the y-intercepts.

The y-intercepts are the minor axis vertices, which are located b units up and down from the center.

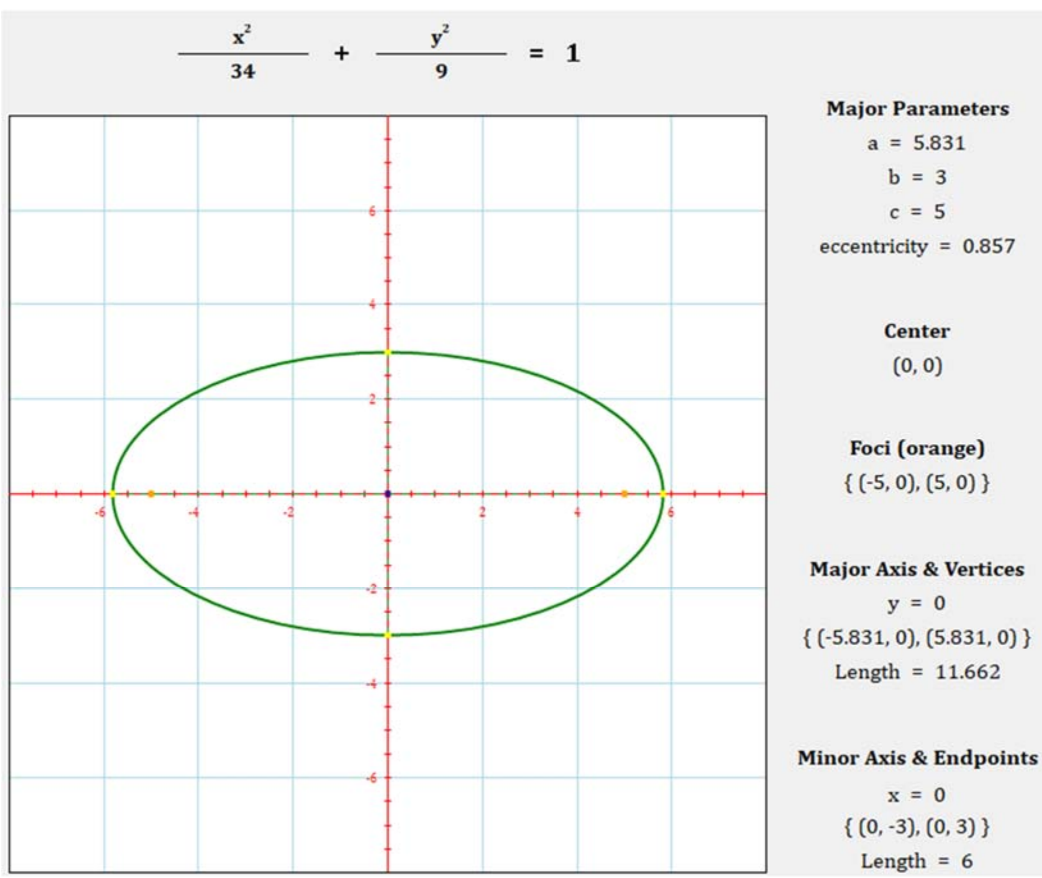
So, we determine that:

- $b=3$
- $c^2=a^2-b^2$ give us $5^2=a^2-3^2$, so $a^2=34$, $a=\sqrt{34}\sim 5.831$

Then, substituting values into the standard form equation gives:

$$\frac{x^2}{34}+\frac{y^2}{9}=1$$

Although a graph is not required, here's what it would look like:



22) Ellipse; endpoints of the major axis: $(12,-4)$ and $(-2,-4)$; endpoints of minor axis: $(5,-8)$ and $(5,0)$

The center of the ellipse is at the midpoint of the major and minor axis endpoints.

- $h = \frac{12+(-2)}{2} = 5,\ k = \frac{-8+0}{2} = -4$

The major axis is in the x-direction (y is constant), so the ellipse has a horizontal major axis.

The standard form for an ellipse with a horizontal major axis is:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

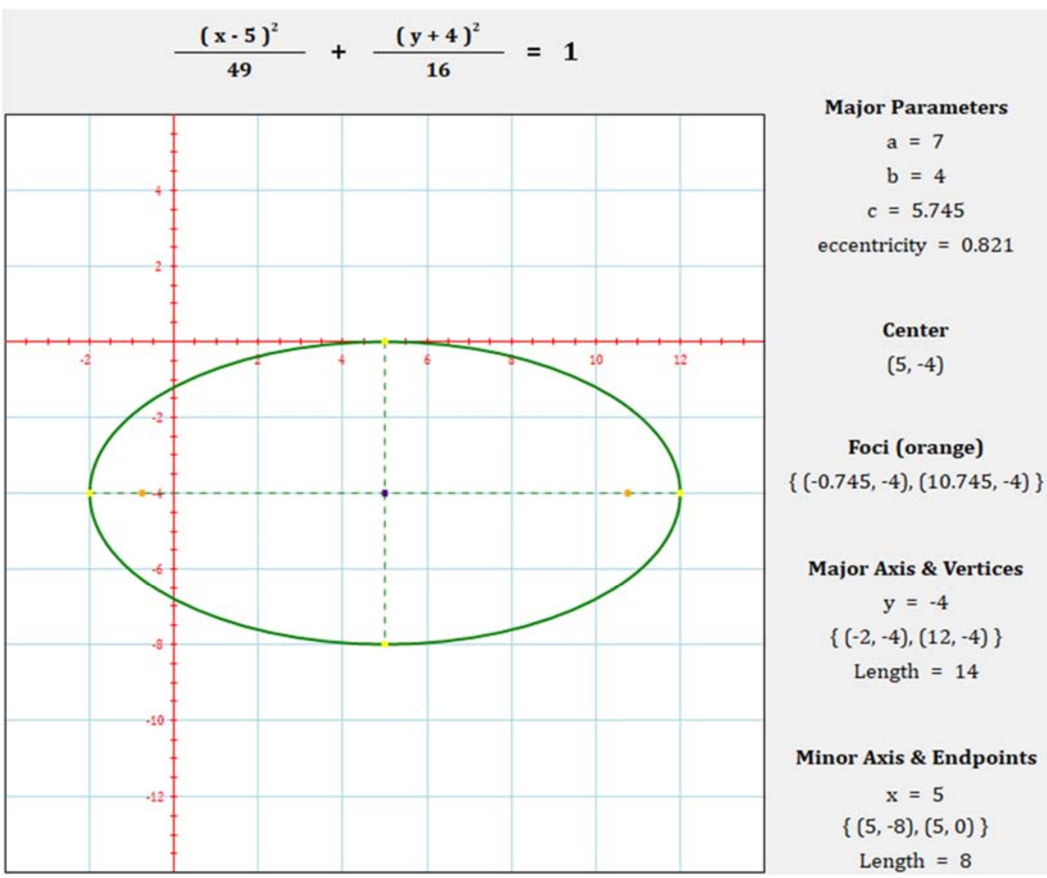
a is half of the length of the major axis, so: $a = \frac{12-(-2)}{2} = 7$. Then, $a^2 = 49$.

b is half of the length of the minor axis, so: $b = \frac{0-(-8)}{2} = 4$. Then, $b^2 = 16$.

Then, substituting values into the standard form equation gives:

$$\frac{(x-5)^2}{49} + \frac{(y+4)^2}{16} = 1$$

Although a graph is not required, here's what it would look like:



23) Parabola; vertex: $(2, -3)$; Focus: $(-1, -3)$

The focus is to the left the vertex on a graph, so the parabola opens to the left. Therefore the parabola has a vertical Directrix. *Note: parabolas always open from the vertex, toward the focus.*

The standard form for a parabola with a horizontal Directrix, according to Blitzer, is:

$$(y-k)^2 = 4p(x-h)$$

Note: There are varying opinions as to what the standard form of a parabola is. The student should use whatever form is preferred by their teacher.

The vertex is located at $(h, k) = (2, -3)$. So, we determine that:

- $h = 2,\ k = -3$

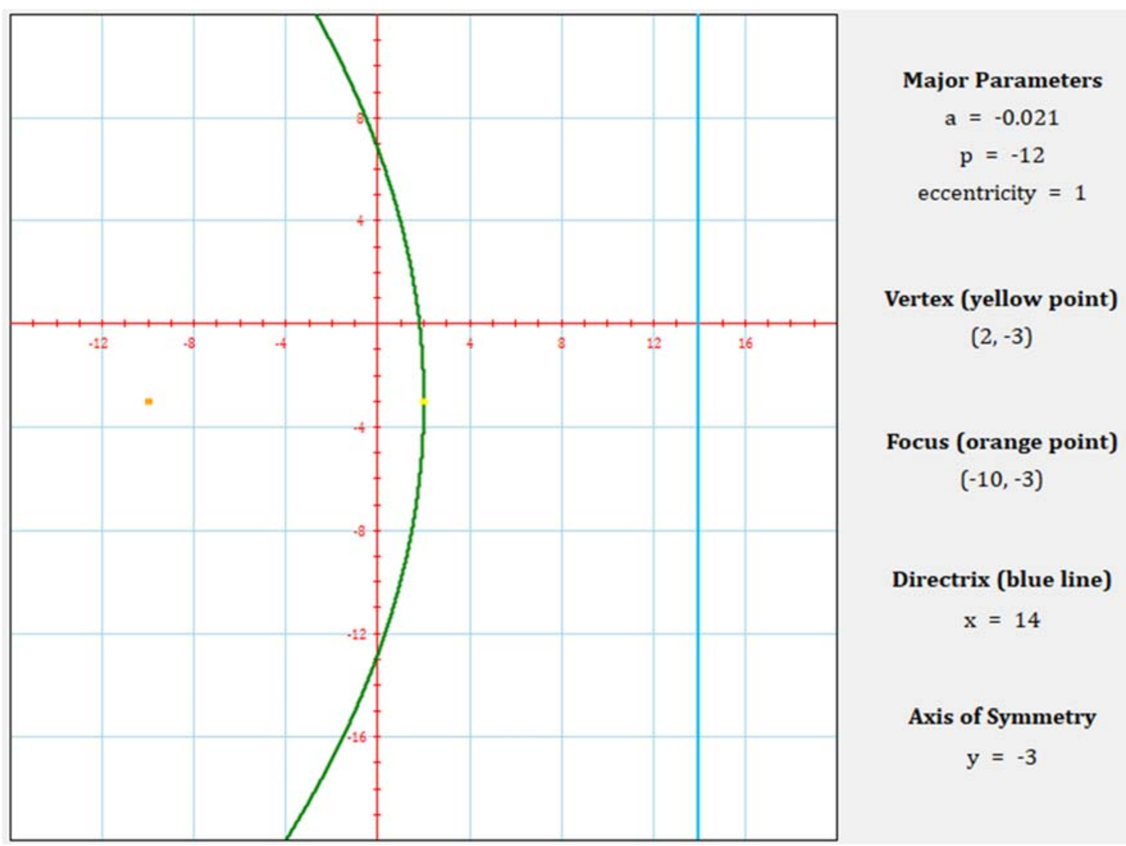
The focus is p units away from the vertex: $p = (focus\ x\ value) - (vertex\ x\ value)$

- $p = -1 - 2 = -3,\ \ 4p = -12$

Then, substituting values into the standard form equation gives:

$$\mathbf{(y+3)^2 = -12(x-2)}$$

Although a graph is not required, here's what it would look like:



24) Hyperbola; foci: $(0,-7),(0,7)$; vertices: $(0,-5),(0,5)$

This hyperbola has foci $(0,\pm 7)$, and therefore has a vertical transverse axis.

The standard form for a hyperbola with a vertical transverse axis is:

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

The center is located halfway between the vertices (and also halfway between the foci):

- $h=\frac{0+0}{2}=0,\ k=\frac{5+(-5)}{2}=0$ $\qquad h=\frac{0+0}{2}=0,\ k=\frac{7+(-7)}{2}=0$

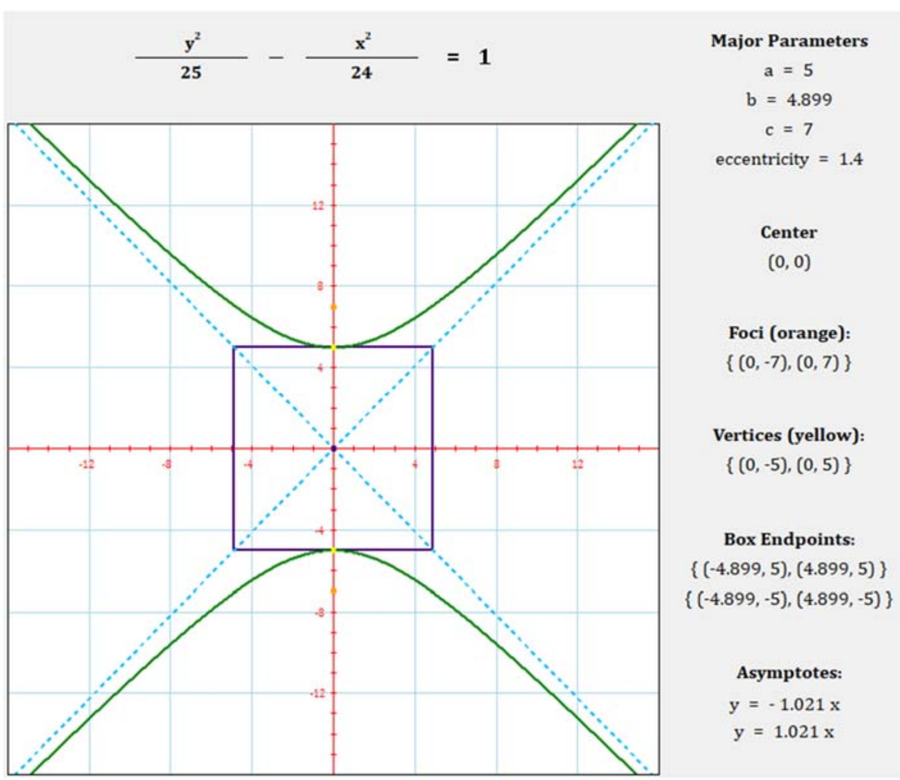
The values of a and b can be determined from the foci and the vertices.

- The vertices are a units up and down from the center**:** $a=5-0=5,\ \ a^2=25$
- The foci are c units up and down from the center**:** $c=7-0=7,\ \ c^2=49$
- Since $c^2=a^2+b^2$ for a hyperbola, we have: $b^2=c^2-a^2=49-25=24$

Then, substituting values into the standard form equation gives:

$$\boldsymbol{\frac{y^2}{25}-\frac{x^2}{24}=1}$$

Although a graph is not required, here's what it would look like:



For #25 – 26, write the equation in standard form for the described conic.

25) Hyperbola; endpoints of transverse axis: $(4,0), (-4,0)$; asymptote: $y = \frac{5}{4}x$

Since the transverse axis endpoints have the same y-value, the hyperbola has a horizontal transverse axis.

The endpoints of the transverse axis are also called the vertices of the hyperbola. The center of the hyperbola is located at (h,k), which is the midpoint of the vertices, so:

- $(h,k) = \left(\frac{-4+4}{2}, \frac{0+0}{2}\right) = (0,0)$
- $h = 0,\ k = 0$

The vertices are a units left and right from the center: $a = 4 - 0 = 4,\ \ a^2 = 16$

Standard form for a hyperbola with a horizontal transverse axis is: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

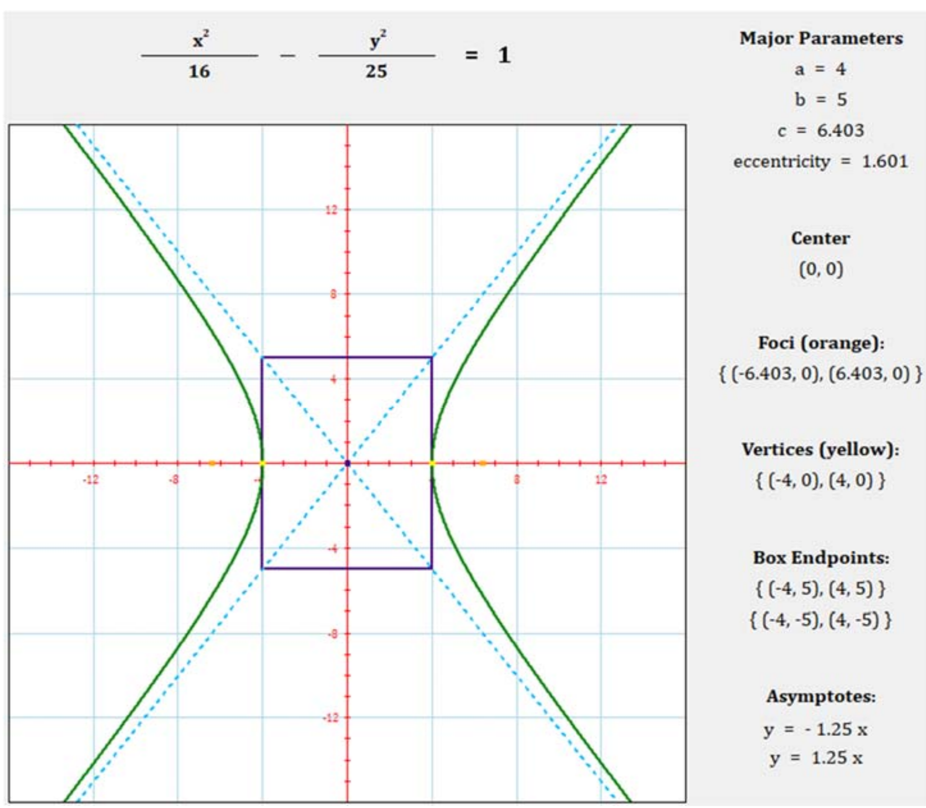
The asymptotes of a hyperbola with a horizontal transverse axis are $y = \pm\frac{b}{a}(x-h) + k$. So:

- $\frac{b}{a} = \frac{5}{4}$ and $a = 4$ so, $b = 5$

Then, the standard form for the hyperbola defined above is:

$$\mathbf{\frac{x^2}{16} - \frac{y^2}{25} = 1}$$

Although a graph is not required, here's what it would look like:



26) Parabola; focus at $(3, -4)$ and directrix at $y = -6$.

The parabola described above has a horizontal Directrix, so it opens up or down.

The focus is above the Directrix, so the parabola opens up. *Note: parabolas always open from the Directrix, toward the focus*.

For a parabola with a horizontal Directrix:

- The vertex is halfway between the focus and the Directrix, so the vertex is:

$$(h, k) = \left(3, \frac{-4 + (-6)}{2}\right) = (3, -5) \quad \Rightarrow \quad h = 3; \ \ k = -5$$

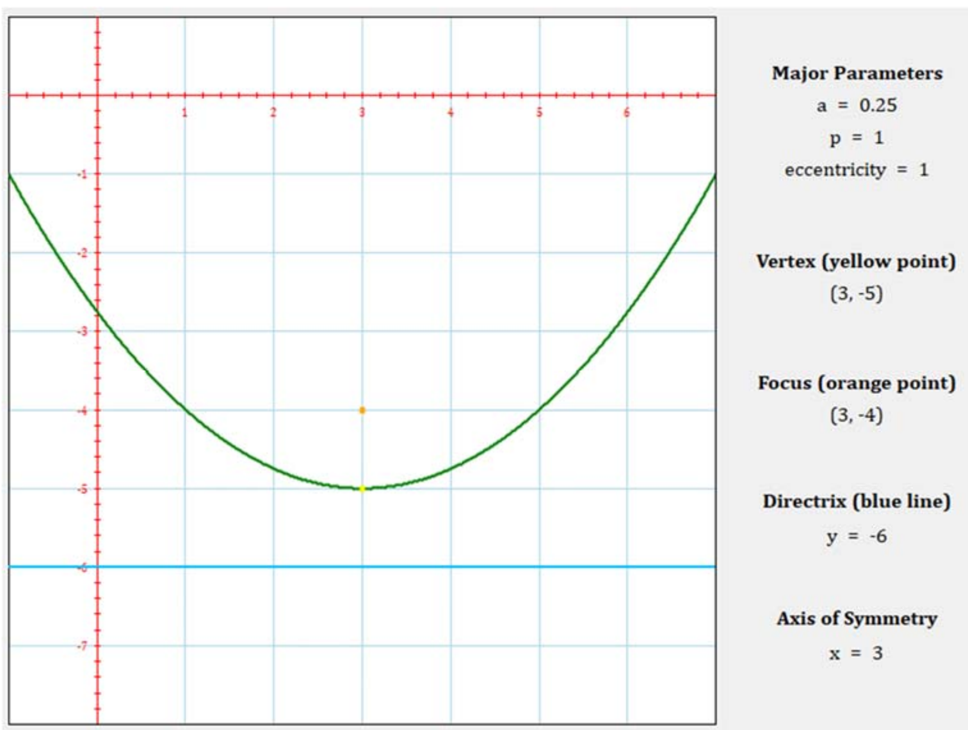
The focus is p units away from the vertex: $p = (focus\ y\ value) - (vertex\ y\ value)$

- $p = -4 - (-5) = 1, \ \ 4p = 4$

We can now write the equation in standard form: $(x - h)^2 = 4p(y - k)$

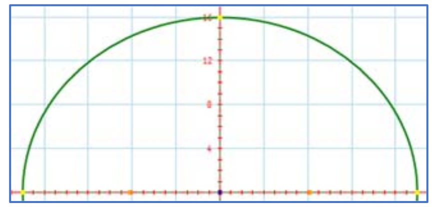
$$\boldsymbol{(x - 3)^2 = 4(y + 5)}$$

Although a graph is not required, here's what it would look like:



27) A tunnel has the shape of a semi-ellipse that is 16ft high at the center, and 36ft across the base. At most how high should a passing truck be, if it is 12ft wide, for it to be able to fit through the tunnel? If needed, round to 3 decimal places.

Graphing the tunnel, we want a minor axis endpoint to be $(0, 16)$, and points on the ellipse: $(-18,0)$ and $(18,0)$.



The tunnel will have a horizontal major axis.

The standard form for an ellipse with a horizontal major axis is:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

The major axis and half of the minor axis are shown in the above figure. Given the points identified above, we see:

$$h = 0, \quad k = 0, \quad a = 36 \div 2 = 18, \quad b = 16$$

The equation of the tunnel, then, becomes:

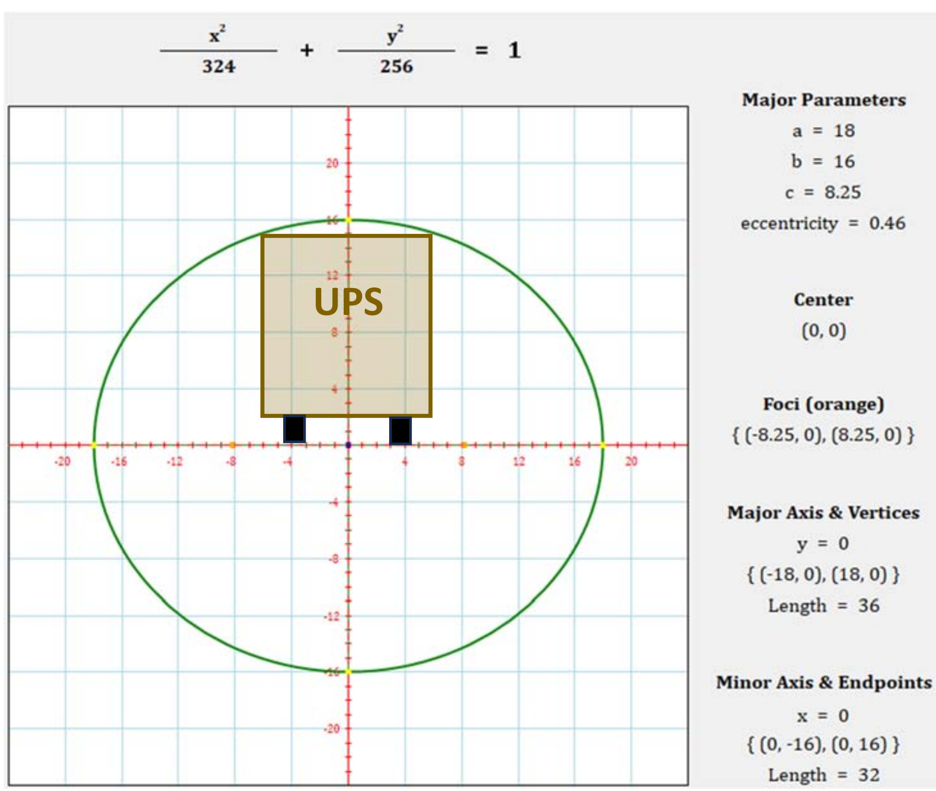
$$\frac{x^2}{18^2} + \frac{y^2}{16^2} = 1 \quad \Rightarrow \quad \frac{x^2}{324} + \frac{y^2}{256} = 1$$

The truck is best able to fit through the middle of the tunnel and is 12 ft wide, so we can let the truck's x values be $x = \pm 6$. This situation is illustrated in the figure below.

Substituting $x = 6$ into the equation, we get the maximum (positive) height of the truck, y:

$$\frac{6^2}{324} + \frac{y^2}{256} = 1 \quad \Rightarrow \quad y = \sqrt{256\left(1 - \frac{6^2}{324}\right)}$$

The truck must have a height of: $y \sim 15.085\ ft.$ or less.



28) A satellite dish has a shape called a paraboloid, where each cross-section is a parabola; since radio signals will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. How far should the receiver be from the vertex, if the dish is 12ft across and 4.5ft deep at the vertex?

A paraboloid is a surface made by spinning a parabola around its axis of symmetry. The cross-section we consider contains the axis of symmetry.

Let the vertex of the parabola be at the origin, with the parabola opening upward. The cross-section will have a horizontal directrix and p will be positive.

The form of the equation will be:

$$(x-h)^2 = 4p(y-k)$$

Since the vertex is at $(0,0)$, $h = k = 0$, and the equation takes the form: $x^2 = 4py$.

The dish is 12 ft across and 4.5 ft deep, a point on the parabola will be $(6, 4.5)$ (see the illustration below). Let's substitute this point into this equation:

$$6^2 = 4p(4.5) \quad \Rightarrow \quad 36 = 18p \quad \Rightarrow \quad p = 2,\ \ 4p = 8$$

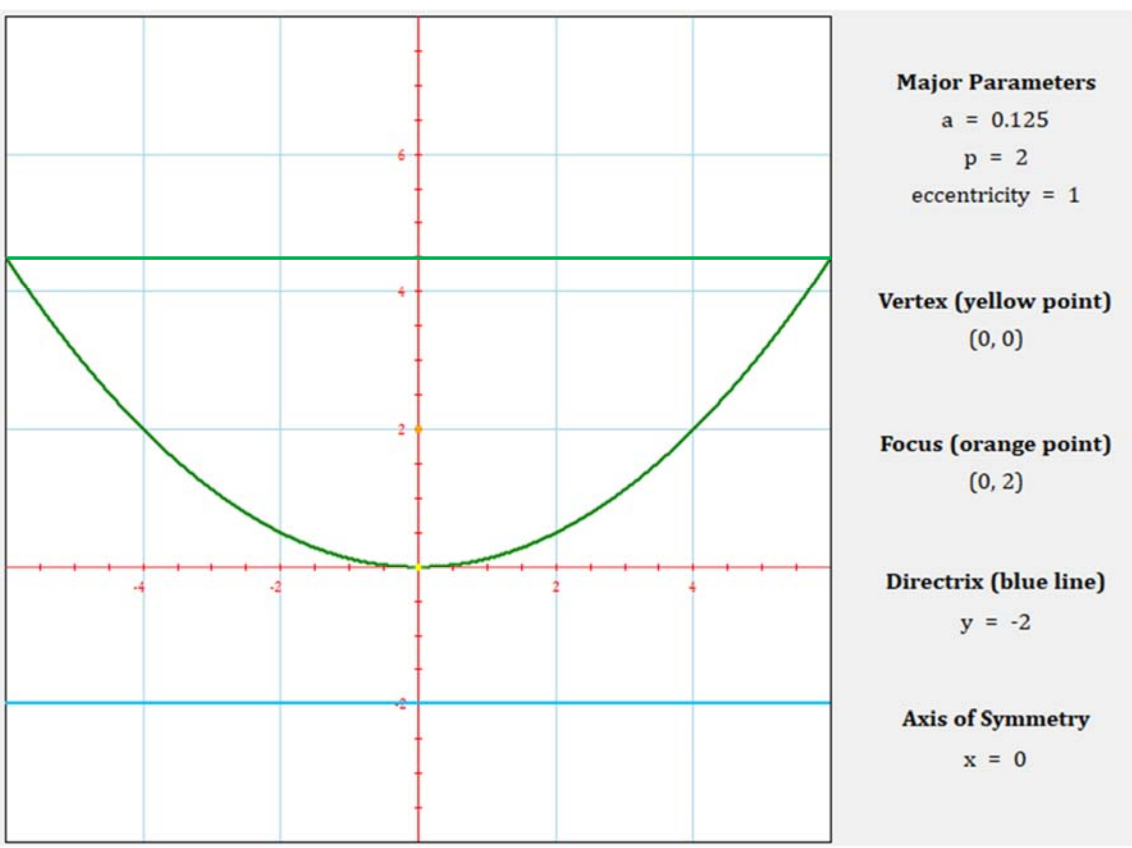
The equation of the cross-section of the paraboloid, then, becomes:

$$x^2 = 8y$$

The *distance between the vertex and the focus of a parabola* is p, and in this problem, $p = 2$ ft. Further, since the dish opens up, the focus is above the vertex (and the directrix is below the vertex).

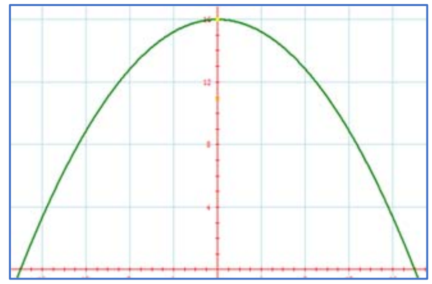
The receiver should be set 2 feet above the vertex.

The figure below illustrates the cross section of the paraboloid satellite dish:



29) (Extra practice) A tunnel has the shape of a parabola that is 16ft high at the center, and 36ft across the base. At most how high should a passing truck be, if it is 12ft wide, for it to be able to fit through the tunnel? If needed, round to 3 decimal places (same as problem 7, but with a parabola).

Graphing the tunnel, we want the vertex to be $(0, 16)$, and points on the parabola: $(-18,0)$ and $(18,0)$.



The tunnel will have a horizontal directrix and it opens down, so p will be negative.

The form of the equation will be:

$$(x-h)^2 = 4p(y-k)$$

Substituting the coordinates of the vertex into this equation, we get: $x^2 = 4p(y-16)$.

The point $(18,0)$ is on the equation of the tunnel, so let's substitute it into this equation:

$$18^2 = 4p(0-16) \quad \Rightarrow \quad 324 = -64p \quad \Rightarrow \quad p = -\frac{81}{16} = -5.0625, \;\; 4p = -\frac{81}{4}$$

The equation of the tunnel, then, becomes:

$$x^2 = -\frac{81}{4}(y-16)$$

The truck is best able to fit through the middle of the tunnel and is 12 ft wide, so we can let the truck's x values be $x = \pm 6$. This situation is illustrated in the figure below.

Substituting $x = 6$ into the equation, we get the maximum height of the truck, y:

$$6^2 = -\frac{81}{4}(y-16) \quad \Rightarrow \quad y = 16 - \frac{6^2 \cdot 4}{81}$$

The truck must have a height of: $y = \frac{128}{9} ft \sim 14.222\ ft$. or less.